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DIELECTRIC HEATING OF LIQUID IN THE REGIME OF TEMPERATURE STRATIFICATION AT A VERTICAL SURFACE UNDER THE CONDITIONS OF NON-STATIONARY RADIATION-CONVECTIVE HEAT TRANSFER

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Abstract
A class of nonlinear problems of liquid dielectric heating in the regime of natural convection near a vertical surface under the conditions of non-stationary radiation-convective heat transfer at microwave influence with a small depth of penetration is studied. These problems are solved using highly effective asymptotic procedures at the successive stages of non-stationary and stationary radiation-convective heat transfer. The non-stationary and stationary parts of solutions are joined by the "vertical coordinate-time" characteristic. The solutions, derived on these principles, are in good agreement with the exact limiting solutions. The error is within the limits of 7%. With a distance from the lower edge of the vertical surface, convective heat transfer changes from the values characteristic of the boundary condition of the second kind to the values characteristic of the boundary condition of the first kind. The rate of this transition depends significantly on the complex parameter of microwave and thermal radiation. An important advantage of solutions to this class of external problems is the fact that even before complex calculations it is possible to perform an exhaustive analysis of the features of the studied processes. Moreover, despite a number of initial simplifications, the latter do not significantly affect the accuracy of results, guaranteeing reliable quantitative information. The developed method can be also extended to the regimes of natural convection with linear dependence of physical properties on temperature, using the Dorodnitsyn transformation. To confirm the adequacy of the constructed mathematical model, an experimental study of stationary radiation-convective heat transfer carried out. Comparison of theoretical and experimental data shows that they are in a good agreement. This again confirms the effectiveness of the developed method for constructing theoretical solutions to the nonlinear problems of natural convection using the asymptotic procedures.
Keywords: dielectric heating, nonstationary heat transfer, convection

Introduction

The historically new type of heating with the help of microwave energy has attracted an increasing number of researchers, technologists, designers and other specialists in expanding the scope of this non-standard heat source in various processes and technologies. First of all, such unique properties of microwave radiation as inertia-free character, concentrating huge power in the right place during a given time period, long-distance transmission in the absence of a heat-carrying medium, almost total conversion of microwave energy into heat, etc. are relevant now [2, 14].

Examples of successful application of microwave radiation are as follows: heat treatment of materials for various purposes; microwave drying of agricultural products; enrichment of ores by microwave exposure; microwave pyrolysis and gasification of coal fuel; microwave dispersion of solid materials, etc. In a number of chemical technologies, the use of microwave radiation accelerates the rate of chemical reactions by tens and hundreds of times.

The problems of dielectric heating of various moving media remain topical at the present stage. Especially they are in demand in the presence of such complicating factors as non-stationarity, complex radiation-convective heat transfer, non-linear heat transfer by heat radiation to the environment, etc.

This paper is devoted to a theoretical study of the problem of microwave heating in the case of natural convective motion of liquid near a vertical surface under the conditions of nonstationary radiation-convective heat transfer.

Problem statement

In this case, the free-convective motion of liquid is caused by heterogeneous heating near a vertical surface and it is determined by the coefficient of volume expansion $\beta_T = -\frac{1}{\rho}\left(\frac{d\rho}{dT}\right)_p$. The boundary-layer model proved to be very fruitful for mathematical description of such transport processes [14]. According to Yang's studies [5], using the perturbation method [17], consideration of the higher-order terms is justified only near the leading edge of the plate at very low temperature gradients, that is, near the boundary of boundary layer initiation. In a laminar flow, the effect viscous dissipation and external pressure gradient is usually insignificant. Due to small temperature differences, variability of the thermophysical properties of liquid can be neglected.

Natural convection of liquid occurs near the vertical surface of a plate with low thermal resistance, heated by absorption of microwave radiation by the near-surface zone due to the small penetration depth. The surface heat flux density q_w applied to liquid is expressed as follows

$$q_w = AS, \qquad (1)$$

where S is flux density of microwave radiation, W/m^2;

A is absorption coefficient of microwave radiation.

This source of heat energy q_w is dissipated in liquid by temperature T_e under the action of convection mechanism and heat radiation into the environment. Below we consider two modifications of the studied system (Figs. 1 and 2). In the first case, thermal radiation passes through diathermic washing liquid (Fig. 1). In the second case, heat transfer from the system filled with dropping liquid to the environment occurs through a vacuum interlayer (Fig. 2). As a result, the goal is to identify the regularities of the influence of microwave and thermal radiation, nonstationarity, vertical coordinate, etc. on convective heat transfer.

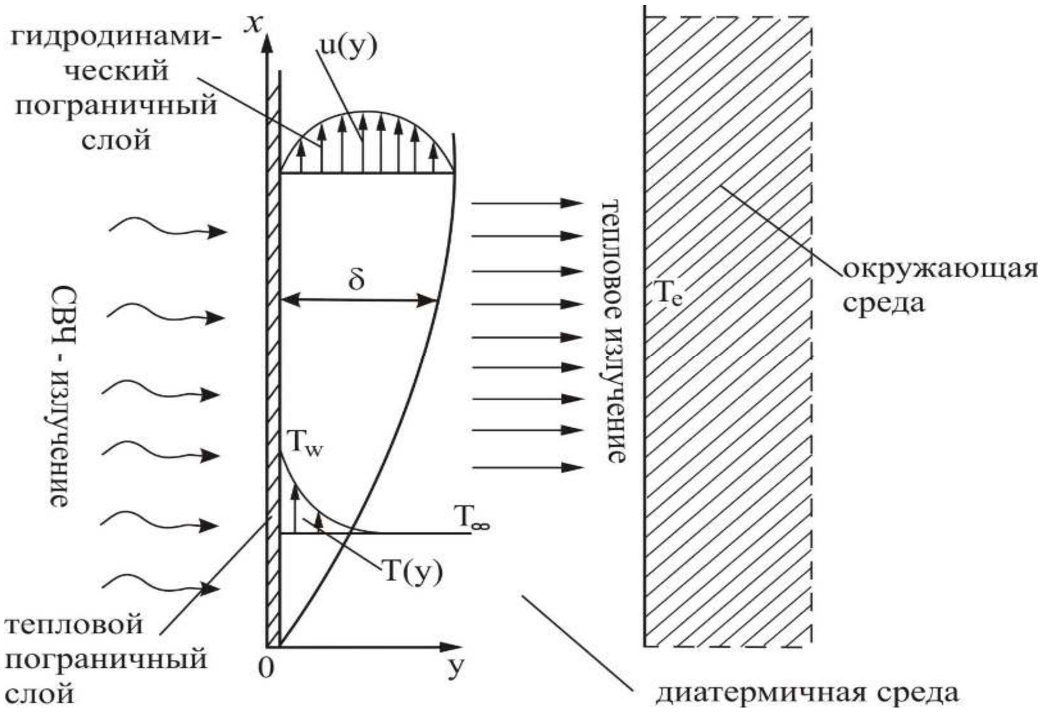

Fig. 1. Microwave heating at natural convection on the heat-radiating surface in a diathermic medium (Hydrodynamic boundary layer, microwave radiation, thermal boundary layer, thermal radiation, ambient medium, diathermic medium)

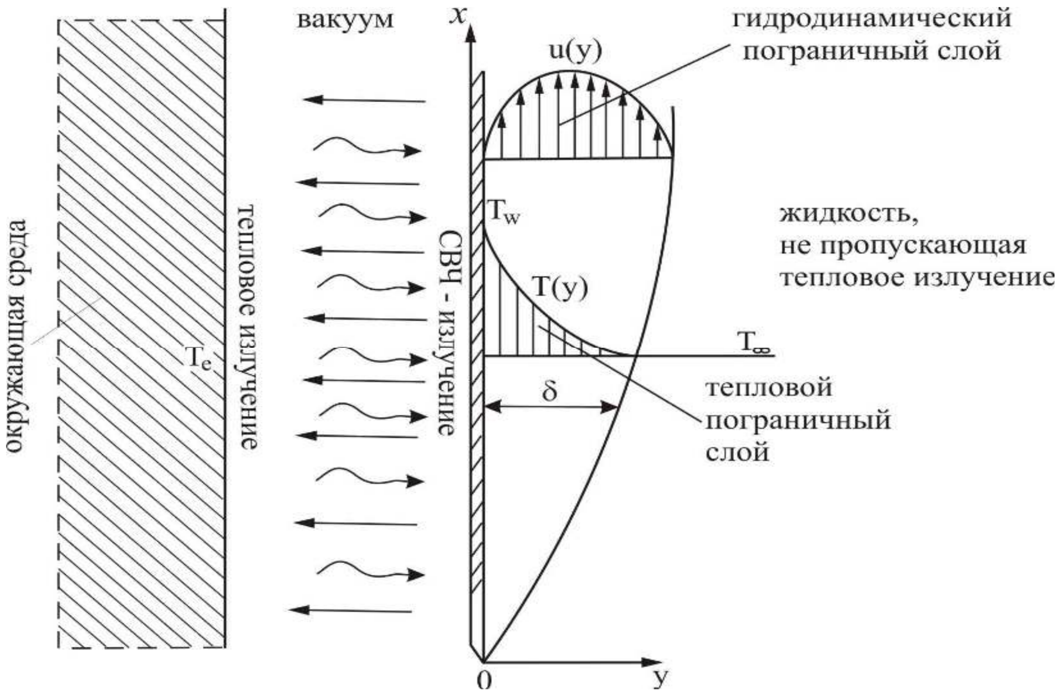

Fig. 2. Microwave heating of liquid, which does not transmit thermal radiation, at natural convection (ambient medium, thermal radiation,vacuum, microwave radiation, hydrodynamic boundary layer, liquid not transmitting thermal radiation, thermal boundary layer)

The stated problem in approximation of the boundary layer is reduced to solution of the following initial-boundary problem in the form of differential laws of conservation of:

mass

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{2}$$

momentum

$$\frac{DU}{dt}\equiv\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)\,, \tag{3}$$

and energy

$$\frac{DT}{dt}\equiv\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}\,, \tag{4}$$

The solution to system (2)–(4) should meet the following initial and boundary conditions
for y = 0

$$u = v = 0, \tag{5}$$

$$\mu\frac{\partial u}{\partial y}=\tau_w\,, \tag{6}$$

$$-\lambda\frac{\partial T}{\partial y}=q_w-\varepsilon\delta_0\left(T_w^4-T_e^4\right)\equiv q_\kappa\,, \tag{7}$$

for y = δ

$$u = 0, \tag{8}$$

$$\frac{\partial u}{\partial y}=0\,, \tag{9}$$

$$T = T_\infty, \tag{10}$$

$$\frac{\partial T}{\partial y}=0\,, \tag{11}$$

$$t \le 0 \quad T = T_\infty, \tag{12}$$

$$x \le 0 \quad T = T_\infty, \tag{13}$$

$$u = v = 0 \tag{14}$$

Here, x, y are longitudinal and transverse coordinates, m; t is time, s; *u*, *v* are longitudinal and transverse velocity components, m/s; T, T_∞, T_w, T_e are current, outer, surface and environment temperatures, K; q_w, q_c are surface and convective heat flux densities, W/m^2; τ_w is shear stress on the wall, n/m^2; δ is boundary layer thickness, m; g is acceleration of gravity, m/s^2; β_T is coefficient of volumetric expansion.

Now it is impossible to find an exact analytical solution to equations (3) and (7) because of their nonlinear character. The only way is to construct the approximate solutions to system (2)–(14) with a controlled error. The common approach is consideration of transfer process at three successive stages [10]

non-stationary

$$\frac{D}{Dt}\equiv\frac{\partial}{\partial t}\,, \tag{15}$$

transient

$$\frac{D}{dt}\equiv\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial t}\,, \tag{16}$$

and steady

$$\frac{D}{dt}\equiv u\frac{\partial}{\partial x}+v\frac{\partial}{\partial t} \tag{17}$$

At the first stage, the basic transport mechanism is molecular. Such an approximate description of the initial stage of non-stationary radiation-convective heat transfer is used somehow for all approximate analytical solutions [10], and it corresponds to the development of a real process with acceptable accuracy. The transition stage, due to its short duration and complexity, usually “contracts to the point”. This method is widely used in convective heat transfer. At the third stage, the molar transport mechanism plays the main role. This regime will replace the previous one, when perturbations from the bottom edge of the plate reach the considered point. For steady radiation-convective heat transfer, non-stationary transfer becomes less pronounced as compared with the convective one. Even in the case of a relatively rapid change in the wall temperature, one can successfully apply the assumption of the stationary character of radiation-convective heat transfer immediately after passing through the non-stationary stage of the process [7]. The general solution is found after the “joining” the non-stationary and stationary stage along the characteristic passing in the “longitudinal coordinate-time” plane.

Then, it is assumed that the thicknesses of hydrodynamic and thermal boundary layers are also equal. This is possible, since thermal stratification and initiated lift are localized within the temperature boundary layer δ. The thermal-convective flow caused by this force should extend beyond the limits of the thermal boundary layer because of viscosity. However, according to the data of [6], illustrating the thermal and velocity fields, this effect is evident only for very viscous liquids (Pr >> 1). Thus, for most cases, propagation of the flow field beyond the thermal boundary layer is unimportant.

In view of these two circumstances, and in connection with the presence of nonlinearities, initial-boundary problem (2) - (14) will be solved by an approximate method based on the apparatus of effective asymptotic expansions [13]. First, such solutions are very common because of their compactness. Secondly, to increase the accuracy, we can attract the subsequent terms of asymptotic expansion.

1. Non-stationary stage of heat transfer

The problem statement for the non-stationary stage of heat transfer in dimensionless variables takes form

$$\frac{\partial\Theta}{\partial Fo} = \frac{\partial^2\Theta}{\partial Y^2} \quad (18)$$

$$Fo = 0 \quad \Theta = 1 \quad (19)$$

$$Y = 0 \quad \frac{\partial\Theta}{\partial Y} = Ki_w - Sk(\Theta_w^4 - \Theta_e^4) \equiv Ki_\kappa \quad (20)$$

$$Y = \Delta_т \quad \frac{\partial\Theta}{\partial Y} = 0 \quad (21)$$

where $Y = \frac{y}{l}$ is dimensionless transverse coordinate;

$\Theta = \frac{T}{T_\infty}$, $\Theta_w = \frac{T_w}{T_\infty}$, $\Theta_e = \frac{T_e}{T_\infty}$ are dimensionless current, surface and environment temperatures;

$Fo = \frac{at}{x^2}$ is local Fourier number;

$\Delta = \frac{\delta}{l}$ is dimensionless thickness of boundary layer;

$Ki_\kappa = \frac{q_\kappa x}{\lambda T_\infty}$ is local convective Kirpichev number;

$Ki_w = \frac{ASx}{\lambda T_\infty}$ is local surface Kirpichev number;

$Ra = \frac{g\beta_T x^3 (T - T_\infty)}{\upsilon a}$ is local Rayleigh number;

$Gr = \frac{g\beta_T x^3 (T - T_\infty)}{\upsilon^2}$ is local Grashof number;

$Sk = \frac{\varepsilon\sigma_0 T_\infty^3 x}{\lambda}$ is local Stark number;

$Nu = \frac{q_\kappa x}{\lambda(T_w - T_\infty)}$ is local Nusselt number;

$\Pr = \frac{\nu}{a}$ is Prandtl number;

$\beta = \frac{AS + \varepsilon\sigma_0 T_e^4}{\varepsilon\sigma_0 T_\infty^4}$ is complex dimensionless parameter of microwave and heat radiations;

ε is emissivity;

σ_0 is Stefan-Boltzmann radiation constants, W/(m^2·K^4);

ν is coefficient of kinematic viscosity, m^2/s a is coefficient of temperature conductivity, m^2/s;

λ is coefficient of heat conductivity, W/m·K;

We will derive the asymptotics of problem (18) - (21), using the following property of integral Laplace transformation [3]. Low Fourier numbers Fo correspond to high values of Laplace transformation operator s, and conversely, high Fourier numbers are characteristic of low values of Laplace transformation operator s. Let us transfer the system of equations (18) - (21) to the Laplace image space. As a result, we obtain

$$\Theta_L - \frac{1}{s} = \frac{d}{dY}\left(\frac{d\Theta_L}{dY}\right) \quad (22)$$

$$\frac{\partial\Theta_L}{\partial Y} = Ki_{\kappa L}(s) \quad \text{при } Y = 0 \quad (23)$$

$$\frac{\partial\Theta_L}{\partial Y} = 0 \quad \text{при } Y = \Delta_т \quad (24)$$

Here, index “L” is a parameter after Laplace transformation. Since $Ki_\kappa(s)$ is a disturbing factor in this system, then the formal solution to system (22)–(24) is relationship

$$\Theta_L - \frac{1}{s} = Ki_{\kappa L}(s) F(Y, s) \quad (25)$$

where F(Y, s) is transfer function. Then, we will give the expression of this function for low and high values of transformation operator s

1.1. Asymptotics of solutions for low Fo numbers (high s)

At the initial stage of non-stationary heat transfer, the transfer process develops as in a semi-infinite array because of the finite propagation rate of thermal perturbation. Thenб the system of differential equations in images (22) - (24) can be presented as follows

$$\Theta_L - \frac{1}{s} = \frac{d}{dY}\left(\frac{d\Theta_L}{dY}\right) \qquad (22')$$

$$\frac{d\Theta_L}{dY} = Ki_{\kappa L}(s) \quad \text{at } Y = 0 \qquad (23')$$

$$\frac{d\Theta_L}{dY} = 0 \quad \text{at } Y \to \infty \qquad (24')$$

The solution to the system of equations (22')–(24') will be written as (25), where the transfer function F(Y, s) is presented as an asymptotic expansion into series by a small parameter, including high values of s.

Then, we obtain

$$F(Y,s) \approx \varphi_1(Y,s)\frac{e^{-\sqrt{s}}}{\sqrt{s}} + \varphi_2\frac{e^{-2\sqrt{s}}}{s} + \ldots + \varphi_n\frac{e^{-n\sqrt{s}}}{s^n} \qquad (26)$$

Substituting (26) into main equation (22') and equations of boundary conditions (26) and (27), equating the terms of the same power with the exponent, we obtain the following chain of equations for determining φ_1, φ_2, φ_3

$$\varphi_1''(Y,s) - s\varphi_1(Y,s) = 0 \quad \text{(a)}$$

$$\varphi_2''(Y,s) - s\varphi_2(Y,s) = 0 \quad \text{(b)}$$

$$\varphi_1'(0,s) = \sqrt{s}\,,\ \varphi_1'(\infty,s) = 0 \quad \text{(c)}$$

$$\varphi_2'(o,s) = 0\,,\ \varphi_2'(\infty,s) = 0 \quad \text{(d)}. \qquad (27)$$

Naturally, for high s, the main role belongs to the first term of expansion. The weight of subsequent terms is constantly decreasing. Then, for the known expressions φ_1, φ_2, φ_3, we can write down the solution for the temperature field image

$$\Theta - \frac{1}{s} \approx Ki_{\kappa L}(s)e^{-\sqrt{s}(Y+2)} + \ldots \qquad (28)$$

Returning to the space of originals, using the convolution theorem, we can express the temperature function as

$$\Theta - 1 \approx \int_0^{Fo} Ki_\kappa(\eta)\,\frac{(Y+2)}{\sqrt{\pi(Fo-\eta)}}\exp\left[-\frac{(Y+2)}{4(Fo-\eta)}\right]d\eta + \ldots \qquad (29)$$

Since Ki_κ under the integral includes nonlinearity in the form of the Stefan-Boltzmann law, it is possible to obtain only an approximate solution to equation (29). To this end, we approximate $Ki_\kappa(\eta)$ in a vicinity of $\eta \approx Fo$ by Taylor series

$$Ki_\kappa(\eta) \approx Ki_\kappa(Fo) + (\eta - Fo)Ki_\kappa'(Fo) + \ldots \qquad (30)$$

where ' is time derivative. After integrating (30), we obtain the expression for calculation of the temperature field with consideration of two terms of asymptotic expansion

$$\Theta(Y,Fo) - 1 \approx Ki_\kappa(Fo)\sqrt{\pi Fo}\,erfc\left(\frac{Y+2}{2\sqrt{Fo}}\right) + Ki_\kappa'(Fo)\frac{(Y+2)}{2\sqrt{\pi}} \times$$
$$\times\left\{2\sqrt{Fo}\exp\left[-\frac{Y+2}{4\sqrt{Fo}}\right]^2 - (Y+2)\sqrt{\pi}\,erfc\left(\frac{Y+2}{2}\sqrt{Fo}\right)\right\} + \ldots \qquad (31)$$

Here, special functions erfc and ierfc relate to the Gauss error function.

According to (31), every next term has a higher order of smallness.

Then, confining ourselves to the first term, we obtain an algebraic equation of the fourth degree with respect to Θ_w

$$\frac{\Theta_w - 1}{Ki_w - Sk(\Theta_w^4 - \Theta_e^4)} \approx \sqrt{\pi Fo}\,erfc\frac{1}{\sqrt{Fo}} + \ldots \qquad (32)$$

whose real positive root is expressed in radicals.

To derive the solutions of increased accuracy, we can take into account the next term of asymptotic expansion with derivative Ki'(Θ_w). Knowing the temperature field (31), we can write down the Nusselt number

$$Nu(Fo) \approx \frac{1}{\sqrt{\frac{Fo}{\pi}} + 2\sqrt{Fo} + erfc\frac{1}{\sqrt{Fo}} - \frac{Fo}{2} - \frac{\left(\frac{Fo}{3\pi} + \frac{Fo\sqrt{Fo}}{4\pi}\right)\frac{dFo}{dKi}}{1 - \sqrt{\frac{Fo}{\pi}}\frac{dKi_\kappa}{d\Theta_w}}} \quad (33)$$

Analyzing solution (33), it can be stated that the Nusselt number decreases with increasing intensity of external heat transfer $\frac{dKi_\kappa}{d\Theta_w}$. Thus, at the initial instants (Fo $\to$ 0), the limiting solutions corresponding to thermal conductivity of a semi-bounded array are obtained

$$Nu(Fo) = \sqrt{\frac{\pi}{Fo}} \quad \text{at} \quad \frac{dKi_\kappa}{d\Theta_w} \to 0 \quad (34)$$

and

$$Nu(Fo) = \frac{1{,}3}{\sqrt{Fo}} \quad \text{at} \quad \frac{dKi_\kappa}{d\Theta_w} \to \infty \quad (35)$$

These results demonstrate that with increasing $\frac{dKi_\kappa}{d\Theta_w}$, the process of convective heat transfer degenerates from the boundary conditions of the second kind to the boundary conditions of the first kind. The error of derived formulas does not exceed 3% of the exact values (Loytsyanskiy, 1962). The case of radiation cooling into a medium of zero temperature at low Fo allows the self-similar solution for both the surface temperature

2. $$\frac{\Theta_w - 1}{\Theta_w} = 2Z + \frac{8}{3}\frac{Z^2}{1+4Z}$$

3. Nusselt number

$$Nu(Fo)\sqrt{\frac{Fo}{\pi}} = \frac{1}{1 + \frac{4Z}{3(1+4Z)}}$$

where $Z = Sk\sqrt{\frac{Fo}{\pi}}\,\Theta_w^3$ is self-simulated variable. We should note that with an increase in Z, both the surface temperature and Nusselt number decrease.

1.2. Asymtotics of solutions for high Fo numbers (low s)

For this case, we will present transfer function F(x, s) in the form of expansion by low parameter s

$$F(\mathrm{Y,s}) \approx \gamma_0(Y,s) + s\gamma_1(Y,s) + s^2\gamma_2(Y,s) + ... \quad (36)$$

Substituting expression (36) to main equation (22) and equating the terms with similar exponents s, we obtain a chain of equations for determination of γ_0, γ_1, γ_2…

$$\frac{d}{dY}\left(\frac{d\gamma_0}{dY}\right) = 0,\ \frac{d}{dY}\left(\frac{d\gamma_1}{dY}\right) = \gamma_0,\ \frac{d}{dY}\left(R\frac{d\gamma_2}{dY}\right) = \gamma_1 \ldots (37)$$

Each equation of (37) requires two boundary conditions for the search of integration constants. We will find the first constant from condition (24)

$$\gamma_0'(Y=\Delta) = 0,\ \gamma_1'(Y=\Delta) = 0,\ \gamma_2'(Y=\Delta) = 0 \ldots (38)$$

The second constant will be determined from integral relationships

$$\int_0^{\Delta_T}\frac{d}{dY}\left(\frac{d\gamma_0}{dY}\right)dY = 0,$$

$$\int_0^{\Delta_T}\frac{d}{dY}\left(\frac{d\gamma_1}{dY}\right)dY = \int_0^{\Delta_T}\gamma_0 dY,$$

$$\int_0^{\Delta_T}\frac{d}{dY}\left(\frac{d\gamma_2}{dY}\right)dY = \int_0^{\Delta_T}\gamma_1 dY \quad (39)$$

Using found expressions for γ_0, γ_1, γ_2 and total solution (25) in images, we can return to the space of originals. As a result, the temperature function with consideration of three terms of asymptotic expansion will take form

$$\Theta-1\approx\int_0^{Fo}Ki_\kappa(\eta)d\eta+Ki_\kappa\left[\frac{(Y-\Delta)^2}{2}-\frac{\Delta^2}{6}\right]+\frac{1}{12}\frac{dKi_\kappa}{dFo}\left[\frac{(Y-\Delta)^4}{2}-(Y-\Delta)^2+\frac{7}{30}\right]+... \quad (40)$$

Assuming in (40) that at Y = 0, $\Theta = \Theta_w$, we obtain the Volterra integral equation of the second kind relative to surface temperature Θ_w. Solving this equation with consideration of two terms, we will finally obtain

$$3Sk^2Fo=\frac{(\Theta_w-1)}{(\beta-\Theta_w^4)^2}+2\int_1^{\Theta_w}\frac{(\Theta_w-1)}{(\beta-\Theta_w^4)^2}d\Theta_w \quad (41)$$

The meaning of complex parameter of microwave and heat radiation β can be explained considering the condition on the plate surface

$$q_w=-\lambda\left.\frac{\partial T}{\partial y}\right|_{y=0}+\varepsilon\sigma_0(T_w^4-T_e^4)\equiv q_\kappa+\varepsilon\sigma_0(T_w^4-T_e^4) \quad (42)$$

At $q_\kappa = 0$, value $\sqrt[4]{\beta}$ is the limiting value of the surface temperature under the given conditions. At non-stationary heat transfer, it varies within $\Theta_w=1\div\sqrt[4]{\beta}$. Let us consider the limiting case of heat transfer at low and high Fourier numbers. At low Fo, thermal resistance of the heater thermal boundary layer is low, then $\Theta_w\to 1$. At high Fo, when thermal resistance of the heater thermal boundary layer is significant, heat transfer occurs predominantly through heat radiation, and $\Theta_w\to\sqrt[4]{\beta}$. Let us estimate a contribution of the integral in solution (41). With this purpose we will use the L'Hospital rule [8]

$$\lim\frac{2\int_1^{\Theta_w}\frac{\Theta_w-1}{(\beta-\Theta_w^4)^2}d\Theta_w}{\frac{(\Theta_w-1)^2}{(\beta-\Theta_w^4)^2}}=\left\{\begin{matrix}\frac{0}{0}\\ \\ \frac{\infty}{\infty}\end{matrix}\right.=\lim\frac{1}{1-\frac{4\Theta_w^3(\Theta_w-1)}{\beta-\Theta_w^4}}\begin{cases}1, & при\quad \Theta_w\to 1\\ 0, & при\quad \Theta_w\to\sqrt[4]{\beta}\end{cases} \quad (43)$$

It follows from (43) that the relative contribution of integral decreases with time from 1 to 0. This behavior also affects the Nusselt number

$$Nu(Fo)\approx\frac{1}{\sqrt{3Fo}}\left[1+\frac{\int_1^{\Theta_w}\frac{(\Theta_w-1)}{(\beta-\Theta_w^4)^2}d\Theta_w}{\frac{(\Theta_w-1)^2}{(\beta-\Theta_w^2)^2}}\right]^{1/2} \quad (44)$$

As a result, non-staionary heat transfer at the boundary conditions of the second kind $Nu\sqrt{Fo}=\sqrt{\frac{2}{3}}$ at low Fo degenerates gradually to $Nu\sqrt{Fo}=\frac{1}{\sqrt{3}}$, and this is typical of the boundary conditions of the first kind at high Fo. It is important to note that these limiting solutions differ from exact ones [16] with an error of up to 7%. We represent the integral, accompanying solutions (41),(43), (44), in elementary functions [8].

$$\int\frac{(\Theta_w-1)d\Theta_w}{(\beta-\Theta_w^4)^2}=\frac{\Theta_w(\Theta_w-1)}{4\beta(\beta-\Theta_w^4)}+\frac{1}{8\beta\sqrt{\beta}}\ln\frac{\sqrt{\beta}+\Theta_w^2}{\sqrt{\beta}-\Theta_w^2}-\frac{3\sqrt[4]{\beta}}{16\beta^2}\left[\ln\frac{\sqrt[4]{\beta}+\Theta_w}{\sqrt[4]{\beta}-\Theta_w}+2arctg\frac{\Theta_w}{\sqrt[4]{\beta}}\right] \quad (45)$$

The solutions take the simplest form at non-stationary cooling of liquid by radiation into the environment of zero temperature. Here, $\beta = 0$ and previous solutions (41), (44) are simplified to form

$$3Sk^2Fo \approx \frac{1}{21} + \frac{1 + 4\Theta_w - 6\Theta_w^2}{21\Theta_w^8} \quad (46)$$

$$Nu\sqrt{3Fo} = \left[\frac{6\Theta_w - 7\Theta_w^2 + \Theta_w^8}{21(1 - \Theta_w)} + 1\right]^{\frac{1}{2}} \quad (47)$$

IThe diagrams of calculations by derived dependences (41), (44) are shown in Figs. 3 and 4. It follows from the diagrams that the parameter of combined effect of microwave and thermal radiation β has a significant effect on the temporal variation of the surface temperature and Nusselt number.

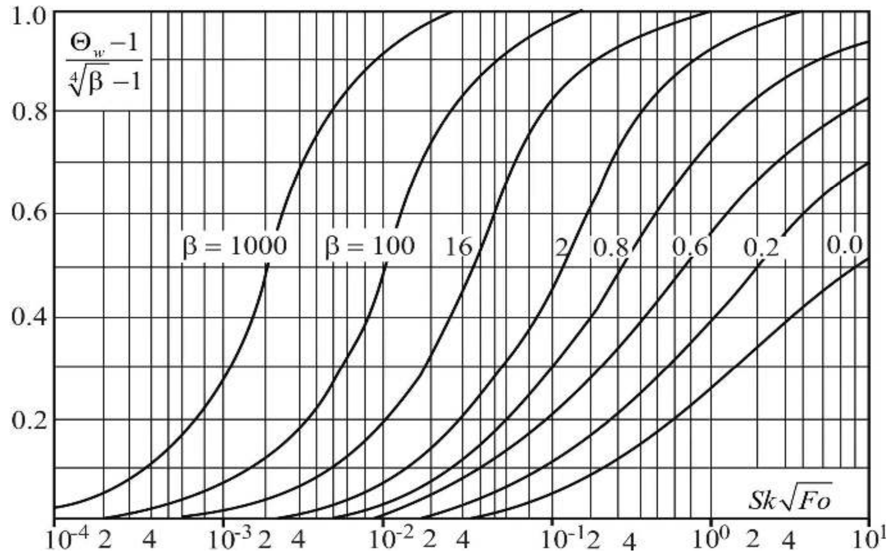

Fig. 3. Effect of complex parameter β on temperature of a vertical surface at natural convection under the conditions of non-stationary heat transfer

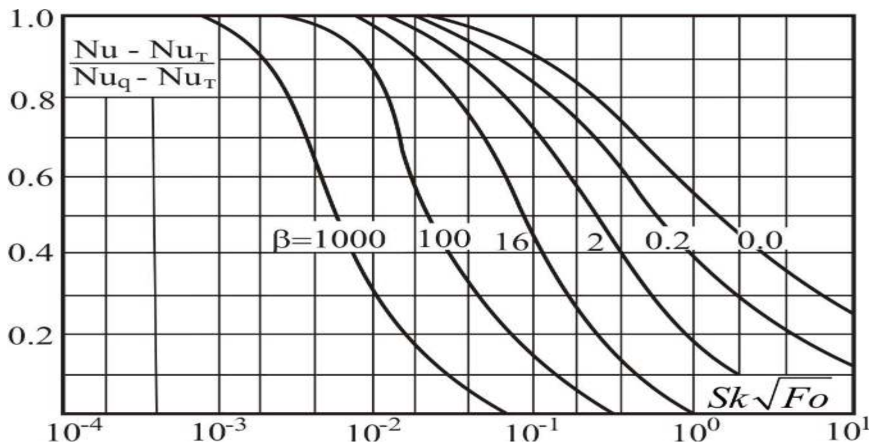

Fig. 4. Change in the Nusselt number at natural convection near the vertical surface under the conditions of non-stationary heat transfer

2. Stationary stage of radiation-convective heat transfer

During the second stage at a predominant effect of convective terms, the determining system of equations looks like:

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \quad (48)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) \quad (49)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2} \quad (50)$$

with boundary conditions

at y = 0

$$u = v = 0 \quad (51)$$

$$-\lambda \frac{\partial T}{\partial y} = q_w - \varepsilon\delta_0 (T_w^4 - T_e^4) \equiv q_\kappa \quad (52)$$

$$u = 0 \quad (53)$$

$$\frac{\partial u}{\partial y} = 0 \quad (54)$$

$$T = T_\infty, \quad (55)$$

$$\frac{\partial T}{\partial y} = 0 \quad (56)$$

A characteristic feature of the system of equations (48) - (56) at its analytic description is, firstly, the interrelationship of equations of momentum and energy transfer. Secondly, it is twice nonlinear because of both momentum conservation equation (49) and equation of boundary condition (52). In this connection, we can seek only its approximate solutions. At the first stage, we linearize the convective terms by introducing the effective rate of transfer

$$U_{эф}(x)\frac{\partial}{\partial x} = u\frac{\partial}{\partial x} + \upsilon\frac{\partial}{\partial y} \quad (57)$$

This linearization is based on Oseen approach [14]. As for the first stage of non-stationary heat transfer, the solution to the thermal problem will be sought using an asymptotic expansion with the procedure of Laplace integral transformation. As a result, asymptotics of solution with consideration of two terms of expansion, is as follows

$$T - T_\infty \cong \frac{a}{\lambda\delta}\int_0^x q_\kappa(\eta)d\eta + \frac{q_\kappa}{\lambda}\frac{3(\delta - y)^2 - \delta^2}{6} \quad (58)$$

where η is integration variable. Using the condition at the external border of the boundary layer at y = δ T = T∞, we obtain from (58)

$$\int_0^x q_\kappa(\eta)d\eta = \frac{q_\kappa\delta^2}{6a} \quad (59)$$

and then the temperature profile

$$T - T_\infty = \frac{q_\kappa\delta}{2\lambda}\left(1 - \frac{y}{\delta}\right)^2, \quad (60)$$

the connection of T_w with $q_к$ and boundary layer thickness follows from this profile at y=0

$$T_w - T_\infty = \frac{q_\kappa\delta}{2\lambda} \quad (61)$$

The solution to equation of motion (49) is derived similarly, and at known temperature (60) with consideration of boundary conditions (51)–(56) it gives the following profile of longitudinal velocity

$$u = \frac{g\beta_T q_\kappa\delta^3}{24\lambda\gamma}\left(-\frac{y^4}{\delta^4} + 4\frac{y^3}{\delta^3} - 5\frac{y^2}{\delta^2} + 2\frac{y}{\delta}\right) \quad (62)$$

We should note that the effective transfer rate U(x) (57) depends in a complex way on y. Pursuing the goal of obtaining the approximate-analytical formulas, we will simplify it slightly, expressing as the average integral one over the boundary layer thickness

$$U_{эф}(x) \cong \frac{\int_0^\delta \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)dy}{\int_0^\delta \frac{\partial T}{\partial x}dy} = \frac{\frac{d}{dx}\int_0^\delta u(T - T_\infty)\,dy}{\frac{d}{dx}\int_0^\delta (T - T_\infty)\,dy} = \frac{\frac{d}{dx}\left(\frac{g\beta_T q_\kappa^2\delta^5}{840\lambda^2\gamma}\right)}{\frac{d}{dx}\left(\frac{q_\kappa\delta^2}{6\lambda}\right)} \quad (63)$$

Turning to variable x, according to (63), we represent integral equation (59) as

$$\int_0^x q_\kappa \frac{d(q_\kappa\delta^2)}{d(q_\kappa^2\delta^5)}dx = \frac{g\beta_T q_\kappa\delta^2}{840\lambda\gamma a} \quad (64)$$

Integral equation (64) allows closed solution

$$\frac{(T_w-T_\infty)^5}{q_\kappa^4}+\frac{5}{3}\int_{T_\infty}^{T_w}\frac{(T_w-T_\infty)^4}{q_\kappa^4}dT_w=\frac{35\upsilon ax}{g\beta_T\lambda^4} \quad (65)$$

We point out as an important circumstance that the integral in (65) is expressed in elementary functions. In the dimensionless form it is

$$\int\frac{(\Theta_w-1)d\Theta_w}{(\beta-\Theta_w^4)^4}=\frac{\Theta_w(\Theta_w-1)^4}{12\beta(\beta-\Theta_w^4)^3}+2\sqrt[4]{\beta}\,\frac{77-270\sqrt{\beta}-7\beta}{512\beta^4}\,arctg\frac{\Theta_w}{\sqrt[4]{\beta}}+$$
$$+\frac{21\beta^2\Theta_w+7\beta\Theta_w^5-128\beta^2+486\beta\Theta_w^3-270\Theta_w^7-400\beta\Theta_w^2+240\Theta_w^6+121\beta\Theta_w-77\Theta_w^5}{384\beta^3(\beta-\Theta_w^4)^2}+ \quad (66)$$
$$+\sqrt[4]{\beta}\,\frac{(77+270\sqrt[4]{\beta}-7\beta)}{712\beta^4}\ln\frac{\sqrt[4]{\beta}+\Theta_w}{\sqrt[4]{\beta}-\Theta_w}-\frac{5}{16\beta^4}\ln\frac{\sqrt[4]{\beta}+\Theta_w^2}{\sqrt[4]{\beta}-\Theta_w^2}$$

Here, $\sqrt[4]{\beta}$ also determine the maximal surface temperature under the given conditions.

Using the L'Hospital rule [8], we can estimate the contribution of integral (66) in solution (65) for the limiting cases. Near the leading edge of the plate $\Theta_w \to 1$, where convective heat transfer predominates. At a large distance from the leading edge, where radiant heat transfer $\Theta_w \to \sqrt[4]{\beta}$ prevails due to the large thermal resistance of the boundary layer

$$\lim\frac{\frac{5}{3}\int_1^{\Theta_w}\frac{(\Theta_w-1)^4d\Theta_w}{(\beta-\Theta_w^4)^4}}{\frac{(\Theta_w-1)^5}{(\beta-\Theta_w^4)^4}}=\begin{cases}\frac{0}{0}\\ \frac{\infty}{\infty}\end{cases}=\lim\frac{5}{3}\frac{\beta-\Theta_w^4}{5(\beta-\Theta_w^4)-16(\Theta_w-1)\Theta_w^3}\begin{cases}\frac{1}{3}, & \Theta_w\to 1\\ 0, & \Theta_w\to\sqrt[4]{\beta}\end{cases} \quad (67)$$

Thus, this integral decreases from 1/3 to 0. Relationship (65) can be presented in dimensionless form

$$\frac{(\Theta_w-1)^5}{(\beta-\Theta_w^4)^4}+\frac{5}{3}\int_1^{\Theta_w}\frac{(\Theta_w-1)^4d\Theta_w}{(\beta-\Theta_w^4)^4}=\frac{35Sk^2}{Ra} \quad (68)$$

It is also possible to determine the Nusselt number

$$Nu=\sqrt[4]{\frac{Ra}{35}}\left(1+\frac{5}{3}\frac{\int_1^{\Theta_w}\frac{(\Theta_w-1)^4d\Theta_w}{(\beta-\Theta_w^4)^4}}{\frac{(\Theta_w-1)^5}{(\beta-\Theta_w^4)^4}}\right)^{1/4} \quad (69)$$

As it was already mentioned, near the lower edge of the plate, $\Theta_w \to 1$. Then, the Nu number corresponds to the boundary conditions of the second kind

$$Nu=\sqrt[4]{\frac{4Ra}{105}} \quad (70)$$

At a distance from the leading edge of the plate, $\Theta_w \to \sqrt[4]{\beta}$ and Nu number corresponds to the boundary conditions of the first kind

$$Nu=\sqrt[4]{\frac{Ra}{35}} \quad (71)$$

The latter indicates that in the regime of stationary radiation-convective heat transfer, as the boundary layer develops, convective heat transfer also degenerates from the boundary conditions of the second kind to boundary conditions of the first kind. These limiting solutions with an accuracy of 4% are consistent with the exact values [16].

It also follows from the limiting solutions that a number of simplifying assumptions in the solution at stage 2 (linearization of convective terms according to (57), simplification of U (x) to the average integral expression over the boundary layer thickness, and keeping a finite number of terms of asymptotic expansion) do not significantly affect the accuracy of final results and they are completely justified. This again indicates effectiveness of the applied asymptotic method for calculating heat transfer in the boundary layer at natural convection. The obtained solutions confirm reliably the qualitative study of R. Cess [1].

In the case of liquid cooling to the medium with zero temperature (β=0), the solutions become significantly simpler

$$\frac{(\Theta_w - 1)^5}{\Theta_w^{16}} + \frac{5}{3}\left(\frac{1}{11\Theta_w^{11}} - \frac{1}{3\Theta_w^{12}} + \frac{6}{13\Theta_w^{13}} - \frac{2}{7\Theta_w^{14}} + \frac{1}{15\Theta_w^{15}}\right) - \frac{1}{77.117} = \frac{35Sk^4}{Ra} \quad (72)$$

$$Nu = \sqrt[4]{\frac{Ra}{35}}\left[1 + \frac{5}{3}\,\frac{\left(\frac{\Theta_w^5}{11} - \frac{\Theta_w^4}{3} + \frac{6\Theta_w^5}{13} - \frac{2\Theta_w^2}{7} + \frac{\Theta_w}{15}\right) - \frac{\Theta_w^{16}}{77\cdot 13\cdot 15}}{(\Theta_w^4 - 1)^5}\right]^{1/4} \quad (73)$$

The obtained solutions (72), (73) are illustrated by the diagrams in Figs. 5, 6. It follows from the figures that with a distance from the leading edge, the Nusselt number changes from the boundary conditions of the second kind to the boundary conditions of the first kind. With an increase in complex parameter β, the process of transition to the boundary conditions of the first kind accelerates.

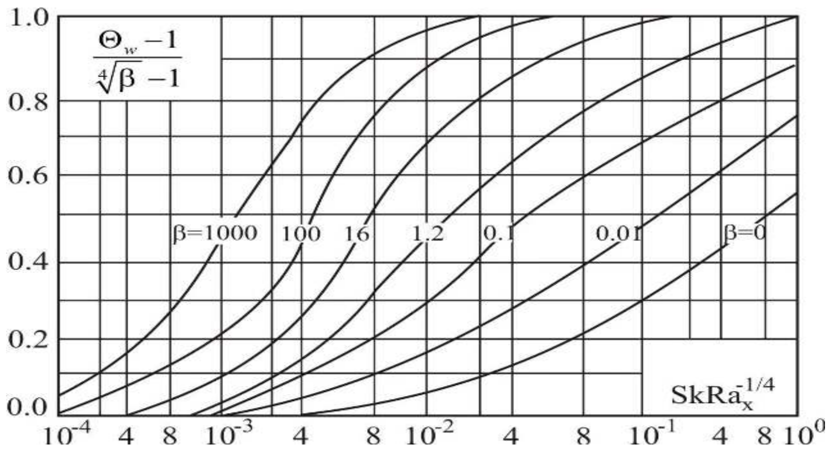

Fig. 5. Change in the stationary temperature of heat-radiating surface along the height under the conditions of natural convection

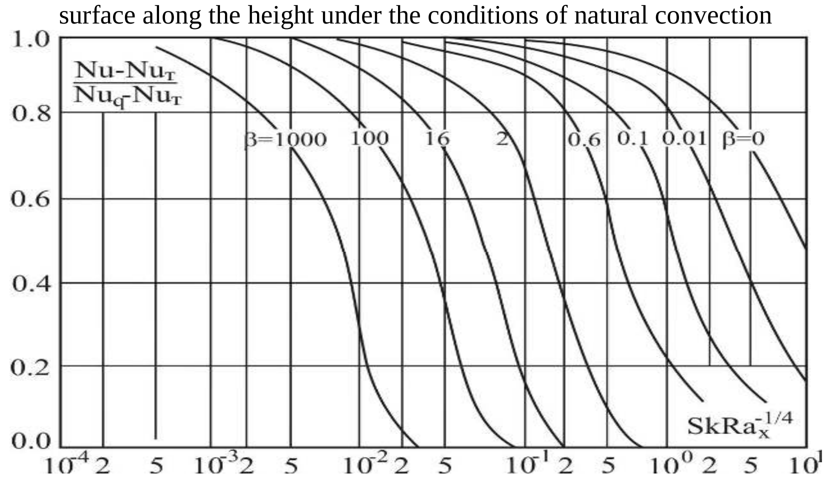

Fig. 6. Change in the stationary Nusselt number along the surface height at natural convection

The above-developed method for solving the nonlinear heat transfer problem at natural convection can be extended to liquids with variable thermal-physical properties subjected to the linear law, using the Dorodnitsyn transformation [4]. Comparing the obtained results with data of R. Cess [1], we note that application of the perturbation method to the problem solved here has a number of fundamental drawbacks. Thus, the existing solutions [1] do not take into account the effect of complex parameter of microwave and thermal radiation β on the process of convective heat transfer. To consider parameter β, it is necessary to derive the higher approximations by R. Cess, which, in turn, makes it necessary to solve the system of complex third-order differential equations, and this is possible only numerically.

3. Harmonization of unsteady and stationary solutions at natural convection

To join obtained solutions (41), (44) and (68), (69), it is necessary to determine the spatial-temporal characteristics, when both solutions are valid. On the boundary between these areas, the solutions must be joined under the condition of equality of temperatures and heat fluxes. The obtained general solution must satisfy initial energy equation (4) and corresponding boundary conditions. To find the boundary of transition from the non-stationary solution to the stationary one, we will use the energy equation in integral form to get the simple connections

$$\frac{\partial}{\partial t}\int_0^{\delta}(T-T_\infty)dy+\frac{\partial}{\partial x}\int_0^{\delta}u(T-T_\infty)dy+=\frac{aq_\kappa}{\lambda} \quad (74)$$

Substituting temperature (60) and velocity (62) profiles into equation (74), we obtain

$$\frac{\partial(q_\kappa\delta^2)}{\partial t}+\frac{\partial}{\partial x}\left(\frac{g\beta_T q_\kappa^2\delta^3}{140\lambda\upsilon}\right)=6aq_\kappa \quad (75)$$

The methods of solution to this equation are presented in hand-book [9]. To solve (75), let us make transformation excluding the boundary layer thickness, according to (61). Using boundary condition (42) and making differentiation, we write down (75) in the form

$$\left\{\frac{2(T_w-T_\infty)}{q_w-\varepsilon\delta_0(T_w^4-T_e^4)}+\frac{(T_w-T_\infty)^2 4\varepsilon\delta_0 T_w^3}{[q_w-\varepsilon\delta_0(T_w^4-T_e^4)]^2}\right\}\frac{dT_w}{dt}+$$
$$+\frac{2g\beta_T\lambda^2}{35\upsilon}\left\{\frac{5(T_w-T_\infty)^4}{\left[q_w-\varepsilon\delta_0(T_w^4-T_e^4)\right]^3}+\frac{3(T_w-T_\infty)\frac{5}{4}\varepsilon\delta_0 T_w^3}{\left[q_w-\varepsilon\delta_0(T_w^4-T_e^4)\right]^4}\right\}\frac{dT_w}{dx}= \quad (76)$$
$$=\frac{3a}{2\lambda^2}\left[q_w-\varepsilon\delta_0(T_w^4-T_e^4)\right]$$

Solving this equation by the method of characteristics [9] gives the system of ordinary differential equations

$$\frac{dt}{\dfrac{2(T_w-T_\infty)}{q_w-\varepsilon\delta_0(T_w^4-T_e^4)}+\dfrac{(T_w-T_\infty)^2 4\varepsilon\delta_0 T_w^3}{[q_w-\varepsilon\delta_0(T_w^4-T_e^4)]^2}}=$$
$$=\frac{dx}{\dfrac{2g\beta_T\lambda^2}{35\upsilon}\left\{\dfrac{5(T_w-T_\infty)^4}{\left[q_w-\varepsilon\delta_0(T_w^4-T_e^4)\right]^3}+\dfrac{12(T_w-T_\infty)^5\varepsilon\delta_0 T_w^3}{[q_w-\varepsilon\delta_0(T_w^4-T_e^4)]^4}\right\}}= \quad (77)$$
$$=\frac{dT_w}{\dfrac{3a}{2\lambda^2}\left[q_w-\varepsilon\delta_0(T_w^4-T_e^4)\right]}$$

The sought solution, describing the motion of the joining boundary between the unsteady and stationary solution, is determined by the differential equation that consists of the first two terms of characteristic system (77)

$$\frac{dt}{2+\dfrac{4\varepsilon\delta_0 T_w^3(T_w-T_\infty)}{q_w-\varepsilon\delta_0(T_w^4-T_e^4)}}=\frac{dx}{\dfrac{2g\beta\,\lambda^2}{35\upsilon}\left\{\dfrac{5(T_w-T_\infty)^3}{\left[q_w-\varepsilon\delta_0(T_w^4-T_e^4)\right]^2}+\dfrac{12(T_w-T_\infty)^4\,4\varepsilon\delta_0 T_w^3}{[q_w-\varepsilon\delta_0(T_w^4-T_e^4)]^3}\right\}} \quad (78)$$

The integral of the last equation with consideration of initial condition *x=0* at *t=0* and known relationship for surface temperature (41) allows determination of a point of transition from the unsteady solution to the stationary one. As a result, the coordinate of stabilization point x_s will be determined along the plate by dependence

$$x_s=\frac{g\beta_T\lambda T_\infty}{35\upsilon(\varepsilon\delta_0 T_\infty^3)^2}\int_0^t\frac{5(\Theta_w-1)^3}{(\beta-\Theta_w^4)^2}\,\frac{1+\dfrac{12}{5}\dfrac{\Theta_w^3(\Theta_w-1)}{\beta-\Theta_w^4}}{1+2\dfrac{\Theta_w^3(\Theta_w-1)}{(\beta-\Theta_w^4)}}\,dt \quad (79)$$

Pursuing the goal of obtaining the analytical assessments of connection $x_s = f(t)$, we will find theses dependences in the limit cases for low ($\Theta_w \to 1$) and high $\Theta_w \to \sqrt[4]{\beta}$. Using the Taylor expansions into series, we will obtain for the initial stage of the process

$$\frac{(\Theta_w-1)^2}{(\beta-\Theta_w^4)^2}+2\int_1^{\Theta_w}\frac{(\Theta_w-1)\,d\Theta_w}{(\beta-\Theta_w^4)^2}\cong 3\left(\frac{\varepsilon\delta_0 T_\infty^3}{\lambda}\right)^2 at \quad (80)$$

Substituting approximate estimate (80) in form

$$\frac{\Theta_w-1}{\beta-1}\cong\frac{\varepsilon\delta_0 T_\infty^3}{\lambda}(3at)^{1/2} \quad (81)$$

into integral relationship (79) and performing integration, we will obtain, limiting ourselves by the first term, dynamics of motion of a stabilization point at the initial stage

$$x_s\cong 0.105\frac{g\beta_T(\beta-1)\varepsilon\delta_0 T_\infty^4}{\lambda\,\mathrm{Pr}}t^{5/2}, \quad (82)$$

at $\Theta_w \to 1$, i.e.,

$$x_s \approx t^{5/2} \quad (83)$$

This relationship is in a good agreement with data of other researchers [15].

The similar estimate can be obtained for time-concluding stage, performing Taylor expansion in a vicinity of $\Theta_w \to \sqrt[4]{\beta}$. Presenting the approximate estimate in form

$$\frac{\sqrt[4]{\beta}-1}{\beta-\Theta_w^4}\cong\frac{\varepsilon\delta_0 T_\infty^3}{\lambda}(3at)^{1/2}-\frac{\sqrt[4]{\beta}}{4\beta}, \quad (84)$$

and introducing it into integral (79), limiting ourselves by the first term, we will obtain the dependence of stabilization point coordinate on time

$$x_s\cong\frac{0.257g\beta_T(T_w-T_\infty)}{\mathrm{Pr}}t^2, \quad (85)$$

at $\Theta_w \to \sqrt[4]{\beta}$, i.e.,

$$x_s \approx t^2$$

These data agree well with [15]. Solutions (82), (85) show that the time of steady-state regime beginning decreases with increasing intensity of thermal radiation, that is, the velocity of characteristic increases as it moves away from the leading edge of the plate.

4. Experimental verification of the theoretical model of radiation-convective heat transfer at natural convection

In order to check the admissibility of the chosen simplifications in constructing the mathematical model and accuracy of theoretical results, we carried out a program of experimental studies on stationary natural convection near a vertical surface. Theoretical solutions of this problem, presented in Sections 1, 2, made it possible to express the determined parameters of the stationary radiation-convective heat transfer as a function of two determining complexes, namely:

$$\frac{\Theta_w - 1}{\sqrt[4]{\beta} - 1} = f_1(\beta, SkRa^{-1/4}) \qquad (86)$$

$$NuRa^{-1/4} = f_2(\beta, SkRa^{-1/4}) \qquad (87)$$

The detailed description of experimental design, selected diagnostics, experimental techniques, measurement schemes, processing of experimental data and their generalization, development of a laboratory setup, and estimation of experimental error are the subject of separate publication of the authors.

The theoretical and experimental values of the surface temperature and Nusselt number at natural convection are compared in Figs. 7 and 8. In these experiments, the maximal theoretical measurement error by dimensionless temperature was within 5.6% ≤ Ro ≤ 7.0%, and by Nusselt number, it was within 9.0% ≤ PNu ≤ 12.1%. The diagrams show the reliable correspondence of theoretical and experimental results with an error not exceeding the maximal theoretical values. The figures also show that the effect of heat radiation leads to degeneration of the Nusselt number from the values corresponding to the boundary conditions of the second kind to the values characteristic for the boundary condition of the first kind. The rate of this transition is determined by complex parameter of microwave and thermal radiation β. The experiment confirms the adequacy of the mathematical formulation of the problem of dielectric heating of liquid in the regime of natural convection under non-stationary radiation-convective heat transfer, ensures the validity of simplifications introduced at theoretical analysis, and guarantees the effectiveness of the developed method for solving the nonlinear problems of such complex heat transfer.

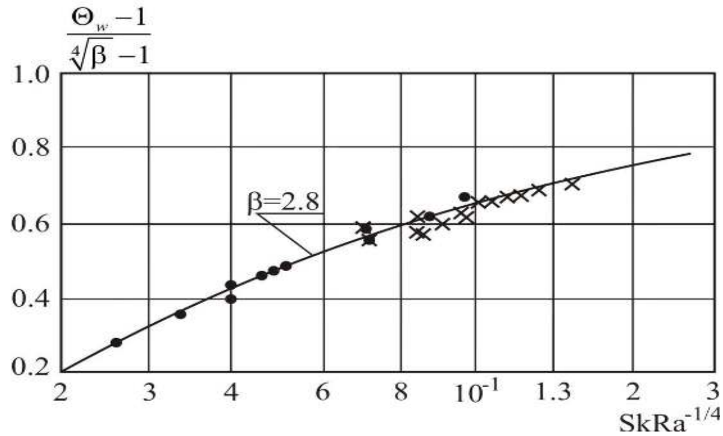

Fig. 7. Comparison of theoretical (solid line, relationship (68)) and experimental results (× – current study, • – data of [11]) on distribution of surface temperature under the conditions of stationary natural convection at β=2.8

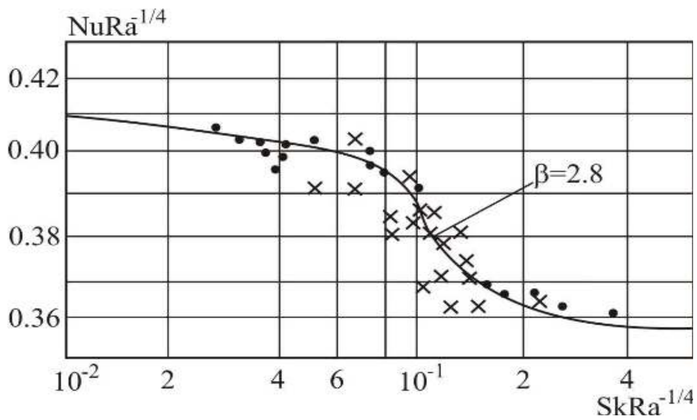

Fig. 8. Comparison of theoretical (relationship (69) – solid line) and experimental results (× – current study, • – experiments of [11]) on the Nusselt number at stationary natural convection, β=2.8

Conclusion

1. A class of nonlinear problems on liquid heating at natural convection under the influence of microwave energy with a small depth of penetration under the conditions of non-stationary radiation-convective heat transfer is investigated. At theoretical analysis, construction of approximate solutions using asymptotic procedures was the most fruitful.

2. In order to obtain the calculated dependences on the temperature fields and Nusselt numbers at natural convection near the vertical surface, an approximate idea of complex heat transfer as a combination of two successive stages (non-stationary and steady) was introduced. Based on this approach, using the asymptotic expansions, the solutions for the indicated stages were derived and compared for the limiting values of the regime parameters. The solutions for non-stationary and stationary heat transfer were joined by the “vertical coordinate-time” characteristic.

3. The developed method made it possible to perform a detailed analysis of complex heat transfer even before complex calculations and reveal the main regularities. It is shown that thermal radiation leads to a change in convective heat transfer from the boundary conditions of the second kind to the boundary conditions of the first kind. The rate of this transition is determined by the complex parameter of microwave and thermal radiation β.

4. To confirm the correct choice of the mathematical model and corresponding simplifications introduced, steady natural convection was studied experimentally. Comparison of theoretical and experimental results gives relatively accurate agreement.

5. The developed method for solving the nonlinear problems of natural convection makes it possible to take into account the linear character of a change in hydrodynamic and thermal-physical properties of heated liquid, introducing the transformation of A.A. Dorodnitsyn.

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ДИЭЛЕКТРИЧЕСКИЙ НАГРЕВ ЖИДКОСТИ В РЕЖИМЕ ТЕМПЕРАТУРНОЙ СТРАТИФИКАЦИИ ОКОЛО ВЕРТИКАЛЬНОЙ ПОВЕРХНОСТИ В УСЛОВИЯХ НЕСТАЦИОНАРНОГО РАДИАЦИОННО-КОНВЕКТИВНОГО ТЕПЛООБМЕНА

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Аннотация
Изучен класс нелинейных задач диэлектрического нагрева жидкости в режиме естественной конвекции около вертикальной поверхности в условиях нестационарного радиационно-конвективного теплообмена при микроволновом воздействии с малой глубиной проникновения. Решения этих задач осуществлено на последовательных стадиях неустановившегося и стационарного теплообмена с применением весьма эффективных асимптотических разложений. Сшивка нестационарной и установившейся частей решений выполнена на характеристике «вертикальная координата – время». Построенные на таких подходах решения находятся в надежном согласии с точными предельными решениями. Погрешность их не выходит за пределы 7%. По мере удаления от нижней кромки вертикальной поверхности происходит изменение конвективного теплообмена от значений, свойственных граничному условию второго рода, до величин, характерных для граничного условия первого рода. Скорость этого перехода сильнейшим образом зависит от комплексного параметра СВЧ и теплового излучений. Важным достоинством решений данного класса внешних задач является то, что еще до проведения сложных расчетов становится возможным провести исчерпывающий анализ закономерностей изучаемых процессов. При этом, не смотря на целый ряд вводимых исходных упрощений, последние существенно не сказываются на точности конечных результатов, гарантируя достоверную количественную информацию. Разработанный метод может быть расширен на режимы естественной конвекции с линейной зависимостью физических свойств жидкости от температуры, применяя преобразование А.А.Дородницына. Для подтверждения адекватности построенной математической модели проведено экспериментальное исследование стационарного радиационно-конвективного теплообмена. Сравнение результатов теоретического и опытного исследования показывает их достаточное соответствие. Это еще раз подтверждает эффективность разработанного метода построения теоретических решений нелинейных задач естественной конвекции с использованием асимптотических процедур.
Ключевые слова: диэлектрический нагрев, нестационарный теплообмен, конвекция

СТАЦИОНАРЛЫ ЕМЕС СӘУЛЕЛЕНУ-КОНВЕКТИВТІК ЖЫЛУ БЕРУ ЖАҒДАЙЫНДА ТІК ҚАБАТТЫҢ ЖАНЫНДА ТЕМПЕРАТУРАЛЫҚ СТРАТИФИКАЦИЯ РЕЖИМІНДЕ СҰЙЫҚТЫҚТЫҢ ДИЭЛЕКТРІК ЖЫЛЫТУ

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Аннотация
Кіші тереңдіктің енуімен микротолқынды әсер кезінде стационарлы емес сәулелену-конвективтік жылу беру жағдайында тік бетіне жақын табиғи конвекция режимінде сұйықтықтың диэлектрлік қызуының сызықты емес мәселелері зерттеледі. Аталған проблемаларды шешу өте тиімді асимптоталық кеңейтуді қолдану арқылы тұрақсыз және стационарлық жылу алмасудың дәйекті

кезеңдерінде жүзеге асырылады. Ерітінділердің стационарлық емес және тұрақты күй бөліктерінің тігісі «тік координаталық уақыт» сипаттамасында орындалады. Осындай көзқарастар бойынша жасалған шешімдер дәл шекті шешімдермен сенімді келіседі. Олардың қателігі 7% -дан аспайды. Тік қабат төменгі жиектен алынады, конвективті жылу алмасу екінші түрдегі шекаралық жағдайға тән мәндерден бірінші түрдегі шекаралық жағдайға тән мәндерге өзгереді. Бұл өтпелі кезеңнің жылдамдығы өте жоғары жиіліктер мен жылу сәулелерінің күрделі параметріне байланысты. Сыртқы проблемалардың осы класындағы шешімдердің маңызды артықшылығы - күрделі есептерге дейін зерттеліп жатқан процестердің заңдылықтарын толық талдау жасауға мүмкіндік береді. Сонымен қатар, бірқатар бастапқы оңайлатулар енгізілгеніне қарамастан, соңғы сандық деректерге кепілдік беретін түпкілікті нәтижелердің дұрыстығына айтарлықтай әсер етпейді.
Әзірленген әдіс А.А. Дородницинның трансформациясын қолдана отырып, сұйықтықтың физикалық қасиеттерінің температураға сызықтық тәуелділігі бар табиғи конвекция режимдеріне дейін кеңейтілуі мүмкін. Құрылған математикалық модельдің барабарлығын растау үшін стационарлық радиациялық-конвективтік жылуды тәжірибелік зерттеу жүргізілді. Теориялық және эксперименттік зерттеулердің нәтижелерін салыстыру олардың жақсы келісілгенін көрсетеді. Бұл асимптотикалық процедураларды қолдана отырып, табиғи конвекцияның сызықты емес міндеттерінің теориялық шешімдерін жасаудың әзірленген әдісінің тиімділігін тағы да растайды.

Түйін сөздер: диэлектрлік жылу, стационарлық емес жылу беру, конвекция